Handout 6


MATH 140 Lab: Section 1
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Note: This handout contains a review for some important things in limits and derivatives.

- If you cannot find the limit for a particular question using any method you studied before, then there is still one possible way to solve that. This method is called L'Hôpital's Rule.
For example: Find $\lim _{x \rightarrow 1}\left(\frac{\ln (x)}{x-1}\right)$.
Solution: L'Hôpital's Rule:
1- Find the derivative for
Numerator(The derivative of natural logarithm of $x$ is $\frac{1}{x}$ ).
2- Find the derivative for Denominator(The derivative of $(x-1)$ is 1 ).
3- $\lim _{x \rightarrow 1}\left(\frac{\ln (x)}{x-1}\right)=\lim _{x \rightarrow 1}\left(\frac{\frac{1}{x}}{1}\right)=\frac{\frac{1}{1}}{1}=1$.
- The derivative of $f$, denoted by $f^{\prime}(x)$, can be written as follows:
$f^{\prime}(x)=\lim _{h \rightarrow 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ provided that the limit exists.
- Pascal Triangle Method for Simplification:

|  | 1 |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 11 |  |  |  |  |
| 121 |  |  |  |  |
| $\begin{array}{lllll}1 & 3 & 3 & 1\end{array}$ |  |  |  |  |
|  |  | 4 | 64 | 1 |
| 1 | 5 | 10 | $0 \quad 105$ | 5 |

## Examples:

$$
\begin{gathered}
(x+y)^{2}=x^{2}+2 x y+y^{2} \\
(x+y)^{4}=x^{4}+4 x^{3} y+6 x^{2} y^{2}+4 x y^{3}+y^{4} \\
(x-y)^{4}=(x+(-y))^{4}
\end{gathered}
$$

- The derivative of $f$, denoted by $f^{\prime}(x)$, can be written in the following three different ways:

$$
\begin{aligned}
& f^{\prime}(x)=\lim _{h \rightarrow 0}\left(\frac{f(x+h)-f(x)}{h}\right) \\
& f^{\prime}(a)=\lim _{h \rightarrow 0}\left(\frac{f(a+h)-f(a)}{h}\right) \\
& \text { OR } \\
& f^{\prime}(a)=\lim _{x \rightarrow a}\left(\frac{f(x)-f(a)}{x-a}\right)
\end{aligned}
$$

## - Important Theorems:

## Derivatives:

1- The derivative of $\sin (x)$ is: $\cos (x)$
2- The derivative of $\cos (x)$ is: $-\sin (x)$
3- The derivative of $\tan (x)$ is: $\sec ^{2} x$
4- The derivative of $\cot (x)$ is: $-\csc ^{2} x$
5- The derivative of $\sec (x)$ is: $\sec (x) \tan (x)$
6- The derivative of $\csc (x)$ is: $-\csc (x) \cot (x)$
7- The derivative of $\ln (\mathrm{x})$ is: $\frac{1}{x}$
8- The derivative of $e^{2 x}$ is $2 e^{2 x}$

## Simplifications:

1- $\cos (A+B)=\cos (A) \cos (B)-\sin (A) \sin (B)$
2- $\sin (A+B)=\sin (A) \cos (B)+\sin (B) \cos (A)$

## Trigonometric Functions:

1- $\sin (0)=0$
2- $\cos (0)=1$
3- $\sin (90)=1$
4- $\cos (90)=0$
5- $\sin (180)=0$
$6-\cos (180)=-1$
7- $\sin (360)=0$
$8-\cos (360)=1$
9- $\sin (30)=\frac{1}{2}$
$10-\cos (60)=\frac{1}{2}$
$11-\tan (45)=1$
12- $\tan (90)=$ Undefined
13- $\sin (45)=\cos (45)=\frac{\sqrt{2}}{2}$


