Handout 10

MATH 172 Lab: Sections 7 and 8

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Note: This handout covers only Alternating Series Test and Taylor Polynomials.

Instruction: Work in groups to solve the following mathematical problems. DON'T AFRAID TO MAKE MISTAKES BECAUSE WE LEARN FROM OUR MISTAKES!

Problem 1: Determine whether the following series diverges, converges conditionally, or converges absolutely:

$$\sum_{\Gamma=1}^{\infty}(-1)^{\Gamma}\frac{\Gamma+1}{\sqrt{\Gamma^3+3}} \rightarrow \boxed{\text{So, it's conditionally convergent.}}$$

Compare $\sum_{\Gamma=1}^{\infty}\frac{\Gamma+1}{\sqrt{\Gamma^3+3}}$ with $\sum_{\Gamma=1}^{\infty}\frac{\Gamma}{\Gamma^{3/2}} = \sum_{\Gamma=1}^{\infty}\frac{1}{\Gamma^{1/2}}$

By p-series test, $p = 1/2 < 1$ diverges

$$\lim_{n\to\infty}\left[\frac{(\Gamma+1)}{\sqrt{\Gamma^3+3}}\cdot\frac{\Gamma^{1/2}}{1}\right] = \frac{\Gamma\cdot\Gamma^{1/2}}{\Gamma^{3/2}} = 1 > 0 \text{ diverges}$$

by limit Comparison Test (LCT).

(i) Alternating

So, it's convergent by Alternating series Test.

(ii) $\lim_{\Gamma\to\infty}\frac{\Gamma+1}{\sqrt{\Gamma^3+1}} = \frac{\Gamma}{\Gamma^{3/2}} = \frac{1}{\Gamma^{1/2}} = \frac{1}{\infty} = 0$

(iii) Assume $f(x) = \frac{x+1}{\sqrt{x^3+3}} \Rightarrow f'(x) = \frac{\left[\frac{\sqrt{x^3+3}\cdot 1}{1} - (x+1)\cdot\frac{(3x^2)}{2\sqrt{x^3+3}}\right]}{(\sqrt{x^3+3})^2} =$

$= \frac{\left[\frac{2(x^3+3)-(x+1)\cdot 3x^2}{2\sqrt{x^3+3}}\right]}{(\sqrt{x^3+3})^2} = \frac{\left[\frac{-x^3+6-3x^2}{2\sqrt{x^3+3}}\right]}{(\sqrt{x^3+3})^2} < 0$ decreasing for $x > 1$.

Problem 2: Find the 4th degree Taylor Polynomial for $w(\zeta) = \ln(\zeta)$ centered at $\zeta = 1$.

$w(\zeta) = \ln(\zeta) \longrightarrow w(1) = \ln(1) = 0$

$w'(\zeta) = \frac{1}{\zeta} = \zeta^{-1} \longrightarrow w'(1) = 1$

$w''(\zeta) = -\zeta^{-2} \longrightarrow w''(1) = -1$

$w'''(\zeta) = 2\zeta^{-3} \longrightarrow w'''(1) = 2$

$w^{(4)}(\zeta) = -6\zeta^{-4} \longrightarrow w^{(4)}(1) = -6$

$$\ln(\zeta) \approx \underbrace{\boxed{(\zeta-1) - \frac{(\zeta-1)^2}{2!} + \frac{2(\zeta-1)^3}{3!} - \frac{6(\zeta-1)^4}{4!}}}_{P_4(\zeta)}$$

Therefore, $\boxed{\ln(\zeta) \approx (\zeta-1) - \frac{(\zeta-1)^2}{2} + \frac{(\zeta-1)^3}{3} - \frac{(\zeta-1)^4}{4}}$

$\boxtimes$