Quiz 4

MATH 172 Lab: Section 8

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Note: This quiz covers only the partial fractions and improper integrals.

Show your work and circle your answers. Neatness and organization count!

Question 1: (2 points) Decompose $\frac{2x^2-5x+2}{x^3+x}$ into partial fractions. Be sure to find the values of any unknown constants.

$x^3+x = x(x^2+1)$

$$\frac{\boxed{2x^2-5x+2}}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{\boxed{A(x^2+1)+(Bx+C)x}}{x(x^2+1)}$$

$2x^2-5x+2 = A(x^2+1)+(Bx+C)x$

$x=0$: $\boxed{2=A}$

$x=1$: $-1 = 2A+B+\cancel{C}$

$x=-1$: $9 = 2A+B-\cancel{C}$

add them together

$8 = 4A+2B \rightarrow$ We know $A=2$, then $8=8+2B \Rightarrow 0=2B \Rightarrow \boxed{B=0}$

So, $-1=2A+B+C$

$-1=4+0+C$

$C=-1-4=-5$

$\boxed{C=-5}$

The final solution is $\frac{2x^2-5x+2}{x^3+x} =$

$= \boxed{\frac{2}{x} - \frac{5}{x^2+1}}$

Question 2: (3 points) Compute the following improper integral by evaluating appropriate limits:

$$\int_0^\infty \frac{1}{\sqrt{x}\, e^{\sqrt{x}}}\, dx$$

Hint: you may need to write the above integral as a sum of two integrals: one from 0 to 1 and the other one from 1 to ∞.

$$\int_0^\infty \frac{1}{\sqrt{x}\, e^{\sqrt{x}}}\, dx = \int_0^1 \frac{1}{\sqrt{x}\, e^{\sqrt{x}}}\, dx + \int_1^\infty \frac{1}{\sqrt{x}\, e^{\sqrt{x}}}\, dx$$

$$= \lim_{t \to 0^+} \int_t^1 \frac{1}{\sqrt{x}\, e^{\sqrt{x}}}\, dx + \lim_{t \to \infty} \int_1^t \frac{1}{\sqrt{x}\, e^{\sqrt{x}}}\, dx$$

$$\int \frac{1}{\sqrt{x}\, e^{\sqrt{x}}}\, dx = \boxed{-2e^{-\sqrt{x}} + C}$$

$$u = \sqrt{x}$$

$$du = \frac{1}{2\sqrt{x}}\, dx \Rightarrow \frac{dx}{\sqrt{x}} = 2du$$

$$\Rightarrow 2\int \frac{1}{e^u}\, du = \frac{2e^{-u}}{-1} + C$$

$$= -2e^{-u} + C$$

$$= \boxed{-2e^{-\sqrt{x}} + C}$$

$$\int_0^\infty \frac{1}{\sqrt{x}\, e^{\sqrt{x}}}\, dx = \lim_{t \to 0^+}\left[-2e^{-\sqrt{1}} + 2e^{-\sqrt{t}}\right] + \lim_{t \to \infty}\left[-2e^{-\sqrt{t}} + 2e^{-\sqrt{1}}\right]$$

$$= -2e^{-1} + 2 + 0 + 2e^{-1}$$

$$= 2(\cancel{-e^{-1}} + 1 + \cancel{e^{-1}})$$

$$= 2 \quad \text{Convergent}$$