Chapter 1

Systems of Linear Equations

In this chapter, we discuss how to solve $n \times m$ systems of linear equations using row operations method. Then, we give an introduction to basic algebra of matrix including matrix addition, matrix subtraction and matrix multiplication. We cover in the remaining sections some important concepts of linear algebra such as linear combinations, determinants, square matrix, inverse square matrix, transpose matrix, Cramer's Rule and Adjoint Method.

1.1 Row Operations Method

First of all, let's start with a simple example about $n \times m$ systems of linear equation.

Example 1.1.1 Solve for x and y for the following 2×2 system of linear equations:

$$\begin{cases} 3x + 2y = 5 \\ -2x + y = -6 \end{cases}$$

Solution: Let's start analyzing this 2×2 system.

First of all, each variable in this system is to the power 1 which means that this system is a linear system. As given in the question itself, 2×2 implies that the number of equations is 2, and the number of unknown variables is also 2.

$$2 \times 2$$

(Number of Equations) × (Number of Unkown Variables)

Then, the unknown variables in this question are x and y. To solve for x and y, we need to multiply the first equation $3x + 2y = 5$ by 2, and we also need to multiply the second equation $-2x + y = -6$ by 3. Hence, we obtain the following:

$$\begin{cases} 6x + 4y = 10 \ldots\ldots\ldots\ldots\ldots..1 \\ -6x + 3y = -18 \ldots\ldots\ldots\ldots.2 \end{cases}$$

By adding equations 1 and 2, we get the following:

$$7y = -8$$

Therefore, $y = \frac{-8}{7}$

Now, we need to substitute the value of y in one of the two original equations. Let's substitute y in the first equation $3x + 2y = 5$ as follows:

$3x + 2\left(\frac{-8}{7}\right) = 5$ is equivalent to

$3x = 5 - 2\left(\frac{-8}{7}\right) = 5 + \left(\frac{16}{7}\right) = \frac{51}{7}$ Then, $x = \frac{17}{7}$

Therefore, $x = \frac{17}{7}$ *and* $y = \frac{-8}{7}$

Hence, we solve for x and y.