* Alternating Series Test:

If the alternating series $\boxed{\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \cdots}$ where $(b_n > 0)$, then the following should be satisfied:

(i) $\boxed{b_{n+1} \leq b_n}$ for all n (decreasing).

(ii) $\boxed{\lim_{n\to\infty} b_n = 0}$.

Hence, the series is $\boxed{\text{convergent}}$.

Example: Investigate: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$.

Solution: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ Converges

(i) Alternating.

(ii) Let $f(x) = \frac{1}{x} \Rightarrow f'(x) = \frac{-1}{x^2} < 0 \Rightarrow f$ is decreasing.

(iii) $\lim_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0$

Hence, the series is convergent. ☑.

* Definition 1: A series $\sum a_n$ is called $\boxed{\text{absolutely convergent}}$ if the series $\sum |a_n|$ is $\boxed{\text{convergent}}$.

* Definition 2: A series $\sum a_n$ is called $\boxed{\text{conditionally convergent}}$ if it's convergent but not absolutely convergent.

*Theorem 1: If a series $\sum a_n$ is absolutely convergent, then it's convergent.

(i.e. $\sum |a_n|$ converges $\Longrightarrow \sum a_n$ converges).

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*Now, let's introduce a method I created, and I want to call it Kaabar-Binary Method for Alternating Series Test as follows:

Binary System (BS)		BS Representation	
ON	ON	1	1
ON	OFF	1	0
OFF	OFF	0	0
OFF	ON	0	1

let's call Convergence as a "C" which represents "ON" state $\Longrightarrow$ "1", while the divergence can be called as a "D" which represents "OFF" state $\Longrightarrow$ "0".

Result:

	$\sum a_n$		$\sum \lvert a_n \rvert$	
	1 C	$\Longleftarrow$	1 C	$\Longleftarrow$ "Absolutely Convergent)
	1 C		0 D	$\Longleftarrow$ "Conditionally Convergent)
	0 D	$\Longrightarrow$	0 D	$\Longleftarrow$ ((Divergent))
Doesn't exist $\Longrightarrow$	0 D		1 C	$\Longleftarrow$ This fails (Inconclusive)

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Example 2: Investigate: $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$.

Solution: $\sum_{n=1}^{\infty} \frac{1}{n} \Rightarrow$ p-series $p=1$ $\boxed{\text{diverges}}$.

(i) Alternating

(ii) $\lim_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0$ $\boxed{\text{Converges}}$

(iii) $f(x) = \frac{1}{x} \Rightarrow f'(x) = \frac{-1}{x^2} < 0 \Rightarrow f$ is decreasing.

Hence, it's a $\boxed{\text{conditionally convergent}}$ series. ☒

Example 3: Investigate: $\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$ using hundont 9 inequalities

Solution: $\sum_{n=1}^{\infty} \left|\frac{\cos(n)}{n^2}\right|$

$$\Downarrow$$

$$|\cos(n)| \le 1$$

$$\Downarrow$$

$$\frac{|\cos(n)|}{n^2} \le \frac{1}{n^2}$$ p-series $p=2>1$ converges

So, by Direct Comparison Test, $\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$ converges. ☒