

* definition: linear inequality is a mathematical statement that has one math expression that is "greater than or equal" or "less than or equal" " $>$ " or " \geq " " $<$ " or " \leq " another.

* There are three properties for inequality:

For any numbers say α , β , and γ .

[1-] If $\alpha < \beta$, then $\alpha + \gamma < \beta + \gamma$

[2-] If $\alpha < \beta$, and if $\gamma > 0$, then $\alpha\gamma < \beta\gamma$

[3-] If $\alpha < \beta$, and if $\gamma < 0$, then $\alpha\gamma > \beta\gamma$

Ex 1 | Solve the following:-

$$-3 \leq 7\psi + 5 < 11$$

Solution: subtract 5 from all of them:

$$-3 - 5 \leq 7\psi + 5 - 5 < 11 - 5$$

$$\frac{-8}{7} \leq \frac{7\psi}{7} < \frac{6}{7}$$

$$\Rightarrow \boxed{\frac{-8}{7} \leq \psi < \frac{6}{7}}$$

\Rightarrow ①

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Linear Inequalities

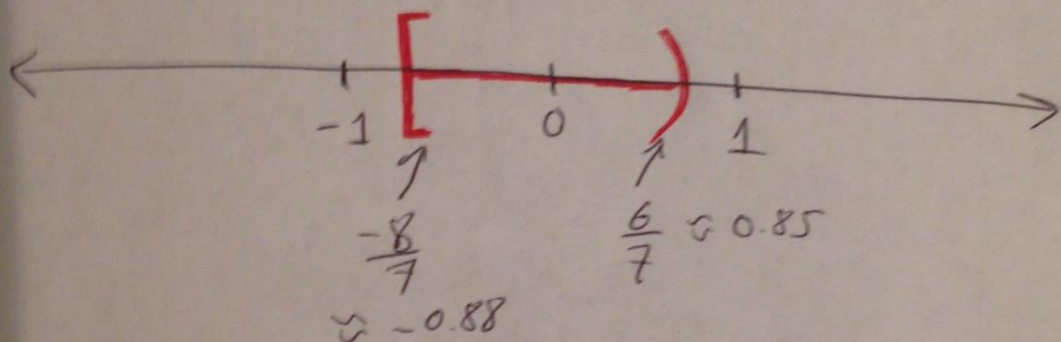
Therefore, the solution is $-\frac{8}{7} \leq \psi < \frac{6}{7}$.

Let's use the interval notation as follows:

$$\left[-\frac{8}{7}, \frac{6}{7} \right)$$

$$-\frac{8}{7} \approx -0.88, \quad \frac{6}{7} \approx 0.85$$

Now, we can also graph our solution as follows:



solution set
Known as
set-builder
notation

Another notation can also be used: Set of solution: $\{ \psi \mid -\frac{8}{7} \leq \psi < \frac{6}{7} \}$

Ex 2) Solve the following:

$$7\delta - 5 < 3\delta + 7$$

Solution: $7\delta - 5 < 3\delta + 7$

$$7\delta - 3\delta < 7 + 5$$

$$\frac{4\delta}{4} < \frac{12}{4}$$

$$\delta < 3$$

The solution of linear inequality.

solution set: $\{ \delta \mid \delta < 3 \}$, Interval notation $\Rightarrow (-\infty, 3)$

Ex 3] Solve the following:

$$-3 \leq 4 - 7\beta < 18$$

Solution:

$$\begin{array}{r} -3 \leq 4 - 7\beta < 18 \\ -4 \quad -4 \quad -4 \end{array}$$

$$\frac{-7}{-7} \leq \frac{-7\beta}{-7} < \frac{14}{-7}$$

$1 \geq \beta > -2 \implies$ So, we can also re-write

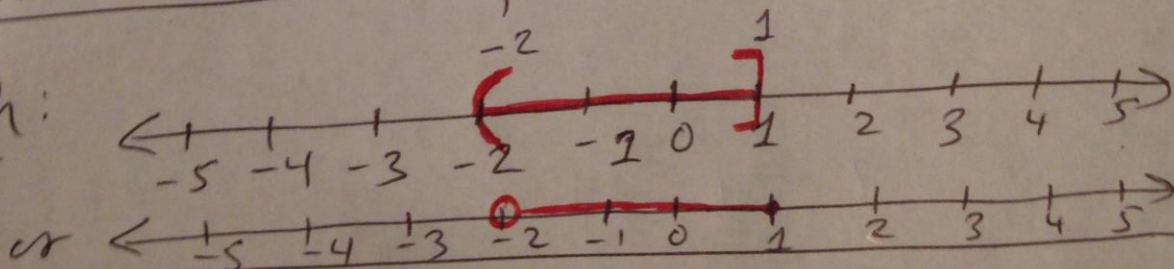
as follows:

$$\boxed{-2 < \beta \leq 1} \leftarrow \text{This is the solution for the linear inequality}$$

Interval Notation: $(-2, 1]$

Set-Builder Notation: $\{\beta \mid -2 < \beta \leq 1\}$

Graph:



Ex 4) Solve the following:

$$-\frac{1}{3} + \frac{5+4x}{2} \geq x - \frac{7}{2}$$

LCD = 6

$$\frac{-2 + 3(5+4x)}{6} \geq \frac{2x-7}{2}$$

$$\frac{-2 + 15 + 12x}{6} \geq \frac{2x-7}{2}$$

$$\frac{13 + 12x}{6} \geq \frac{2x-7}{2}$$

Multiply both sides by 6, we obtain:

$$6 \left(\frac{13+12x}{6} \right) \geq \left(\frac{2x-7}{2} \right) 6$$

$$13 + 12x \geq 6x - 21$$

$$12x - 6x \geq -21 - 13$$

$$6x \geq -34$$

$$x \geq \frac{-34}{6}$$

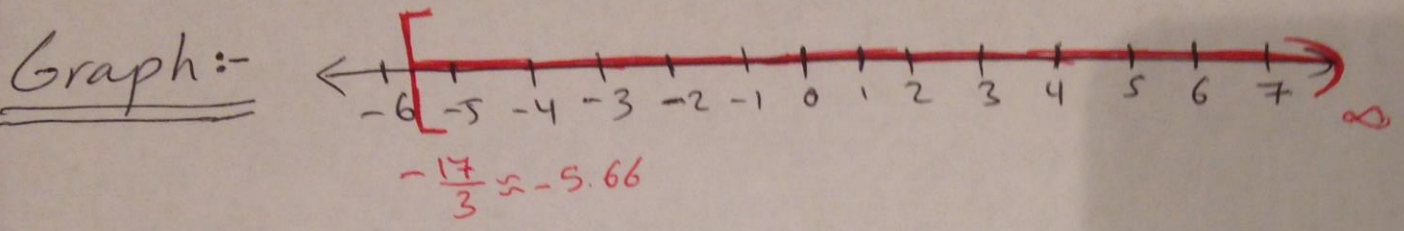
$$\Rightarrow x \geq \frac{-34}{6}$$

$$\text{or } x \geq \frac{-17}{3} \approx -5.66$$

Solution of linear inequality.

Interval notation: $[-\frac{17}{3}, \infty)$

Set-Builder notation: $\{x \mid x \geq -\frac{17}{3}\}$



* Graphs

Ordered pair \Rightarrow (? , !)

How to plot the ordered pair?

Ex 5] Plot the following:

$(-\frac{1}{x}, \frac{1}{y})$

$(\frac{2}{x}, -\frac{1}{y})$

$(-\frac{2}{x}, -\frac{1}{y})$

$(\frac{0}{x}, \frac{1}{y})$

$(\frac{2}{x}, \frac{0}{y})$

