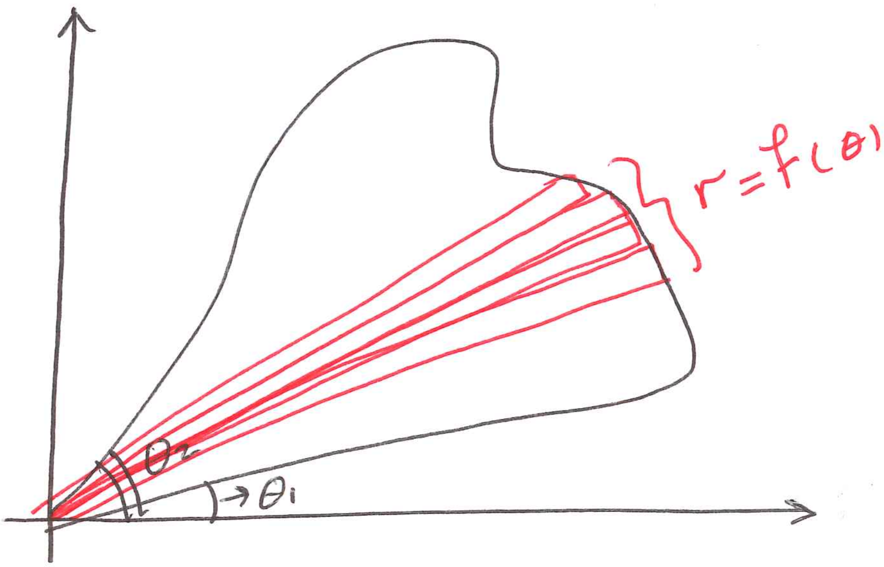
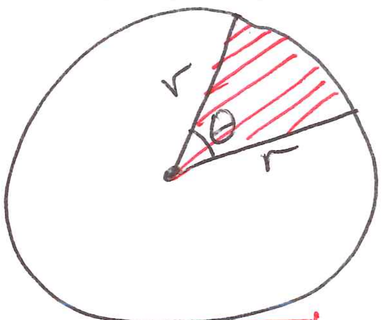
Mohammed K A Kaabar — Area for Polar Coordinates — Dec. 1st, 2015

* Area for Polar Coordinates:





If $\theta = 2\pi$, then the area of entire circle is $\left(\frac{1}{\cancel{2}}\right) r^2 (\cancel{2}\pi) = \boxed{\pi r^2}$

$\boxed{\text{Area} = \frac{1}{2} r^2 \theta}$ ⇐ Area of a sector of a circle

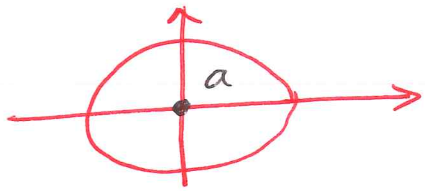
Thus, the area for polar coordinates is:

$$\boxed{A = \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 \, d\theta}$$

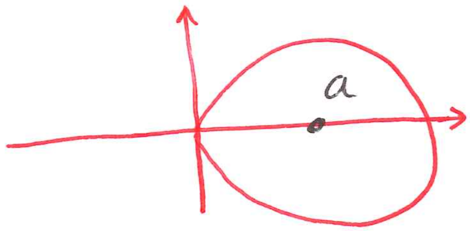
⟹

* Three Common Cases for Polar Coordinates:

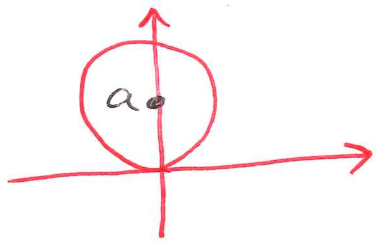
① $r = a$



② $r = a\cos\theta$



③ $r = a\sin\theta$



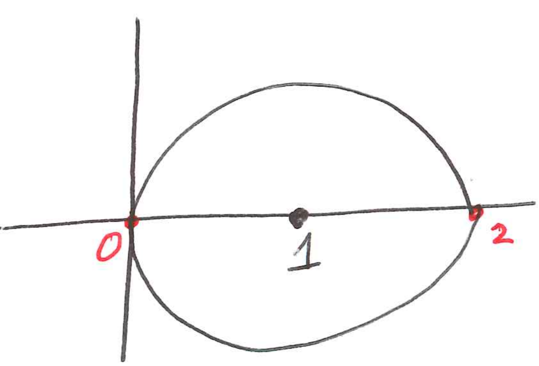
Example: Find the area inside the circle $r = 2\cos\theta$. (Set-up only).

Solution:

$$A = \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 \, d\theta = \frac{1}{2} \int_{\theta_1}^{\theta_2} r^2 \, d\theta$$

From the above 3 common cases for polar coordinates, we know how to draw it as follows:

$\pi r^2 = \pi (1)^2 = \pi$
$\pi r^2 = \pi (0)^2 = 0$



OR apply:

r	2	1	0	-1	-2
θ	0	$\pi/3$	$\pi/2$	$2\pi/3$	π

So, $\boxed{A = \frac{1}{2} \int_{0}^{\pi} (2\cos\theta)^2 \, d\theta}$ $\Rightarrow$

Another Possible Solution:

$$A = 2\left[\frac{1}{2}\int_0^{\pi/2}(2\cos\theta)^2\,d\theta\right]$$

$$= \left(2\cdot\frac{1}{2}\right)\left[\int_0^{\pi/2} 4\cos^2\theta\,d\theta\right]$$

$$= 1\cdot\left[4\int_0^{\pi/2}\cos^2\theta\,d\theta\right]$$

$$= 1\cdot\left[4\int_0^{\pi/2}\left(\frac{1+\cos 2\theta}{2}\right)d\theta\right]$$

$$= 2\int_0^{\pi/2}(1+\cos 2\theta)\,d\theta$$

$$= \int_0^{\pi/2}(2+2\cos 2\theta)\,d\theta$$

$$= \left[2\theta + \frac{\cancel{2}\sin 2\theta}{\cancel{2}}\right]\Bigg|_0^{\pi/2}$$

$$= \left[2\theta + \sin 2\theta\right]\Bigg|_0^{\pi/2} = \left(\cancel{2}\left(\frac{\pi}{\cancel{2}}\right) + \sin \cancel{2}\left(\frac{\pi}{\cancel{2}}\right)\right) - \left(2\cancel{(0)}^{0} + \cancel{\sin(2(0))}^{0}\right)$$

$$= \pi + \cancel{\sin\pi}^{0} - 0$$

$$= \boxed{\pi} \xleftrightarrow{\text{same}} \boxed{\pi}$$

Check your solution: $A = \pi r^2 = \pi(1)^2 = \boxed{\pi}$. Therefore, our solution is correct!! ▪