



Quiz 3

MATH 172 Lab: Section 8

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-Solution-

Note: This quiz covers only the differential equations and integration by parts.

Show your work and circle your answers. Neatness and organization count!

Question 1: (2 points) Find the general solution of the given differential equation:

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

(Hint: General solution means that you need to write it as $y(x)$ as we did in the Differential Equations Lab on Thursday)

$$\Rightarrow \frac{dy}{dx} \times \frac{1}{1+y^2} = \frac{1}{1+x^2} \Rightarrow \frac{1}{1+y^2} dy = \frac{1}{1+x^2} dx$$

$$\Rightarrow \frac{1}{1+y^2} dy - \frac{1}{1+x^2} dx = 0 \Rightarrow \int \frac{1}{1+y^2} dy - \int \frac{1}{1+x^2} dx = \int 0$$

$$\Rightarrow \boxed{\tan^{-1}(y)} - \tan^{-1}(x) = C \Rightarrow \tan^{-1}(y) = \tan^{-1}(x) + C$$

Take tan of both sides

$$\Rightarrow \tan(\tan^{-1}(y)) = \tan(\tan^{-1}(x) + C)$$

$$\Rightarrow \boxed{y(x) = \tan(\tan^{-1}(x) + C) = x + C}$$

Thus, $\boxed{y(x) = x + C}$ where C is a constant. \square

Question 2: (3 points) Evaluate the following integral:

$$\int x \tan^{-1}(x) dx$$

(Hint: Remember $\int \frac{x^2}{1+x^2} dx$ can be written as $\int \frac{(x^2+1)-1}{(x^2+1)} dx$)

By parts: $u = \tan^{-1}(x) \rightarrow dv = x dx$
 $du = \frac{1}{1+x^2} dx \leftarrow v = \frac{x^2}{2}$

$$\int x \tan^{-1}(x) dx = \frac{1}{2} x^2 \tan^{-1}(x) - \frac{1}{2} \int x^2 \left(\frac{1}{1+x^2} \right) dx$$

$$= \frac{1}{2} x^2 \tan^{-1}(x) - \frac{1}{2} \int \frac{x^2}{1+x^2} dx$$

So, $\int x \tan^{-1}(x) dx = \frac{1}{2} x^2 \tan^{-1}(x) - \frac{1}{2} x + \frac{1}{2} \tan^{-1}(x) + C$

Final answer

Since degree (numerator) \geq degree (denominator), then it's long division or use the given hint:

OR long Division

By hint: $\int \frac{x^2+1-1}{1+x^2} dx$

$$= \int \left(\frac{1+x^2}{1+x^2} \right) dx + \int \frac{-1}{(1+x^2)} dx$$

$$= x + \tan^{-1}(x) + C$$

Same answer

$$\int 1 + \left(\frac{-1}{x^2+1} \right) dx = x + \tan^{-1}(x) + C$$

$$\begin{array}{r} 1 \\ x^2+1 \overline{) x^2+1} \\ \underline{\ominus x^2+1} \\ -1 \end{array}$$

$$\frac{x^2}{1+x^2} = 1 + \left(\frac{-1}{x^2+1} \right) = 1 + \frac{-1}{x^2+1}$$