Mohammed Kaabar Polynomials (Part 1) Week 10

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* <u>Definition</u>: A polynomial is a mathematical expression that can be written as follows:

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

where the following:

$n \longrightarrow$ positive integer

$x \longrightarrow$ variable

$a_0, a_1, a_2, \ldots, a_n \longrightarrow$ coefficients of the polynomial (Real numbers).

<u>Note</u>: The highest degree among all other non-zero terms in the above form is the degree of the polynomial.

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* <u>Examples of Polynomials</u>:

① $2x^2 + 5x + 2$

② $2t + 5$

③ $-5t^2 + 2t + 10$

Polynomials (Part 1)

Ex 1) Determine the degree for the following polynomials:

Part a: $-5\alpha^2+15\alpha+2$

Part b: $t^8+5t^7+5t^5+12$

Part c: $\beta^2+13\beta^3+12+\beta$

Solution:

Part a: The degree of this polynomial is $\underline{\underline{2}}$.

Part b: The degree of this polynomial is $\underline{\underline{8}}$.

Part c: The degree of this polynomial is $\underline{\underline{3}}$.

Ex 2) Add the following polynomials:

$$(4x^3+5x^2+1)$$

$$+\ (15x^4+3x^2-x^3+2)$$

Solution:

$$\begin{array}{lcccc} & & 4x^3 & 5x^2 & 1 \\ + & 15x^4 & -x^3 & 3x^2 & 2 \\ \hline & \end{array}$$

$$\boxed{15x^4+3x^3+8x^2+3}$$

Ex 3) Subtract the following Polynomials:

$$(2\alpha^7 + 5\alpha^5 + 10\alpha^3 + 5\alpha^2 + 12)$$

$$-\ (15\alpha^6 - \alpha^5 + 12\alpha + 10 - 3\alpha^3)$$

Solution:

$$\begin{array}{cccccccc} 2\alpha^7 & 0 & 5\alpha^5 & 10\alpha^3 & 5\alpha^2 & 0 & 12 \\ \ominus\ 0 & 15\alpha^6 & -\alpha^5 & -3\alpha^3 & 0 & 12\alpha & 10 \end{array}$$

$$= \boxed{2\alpha^7 - 15\alpha^6 + 4\alpha^5 + 7\alpha^3 + 5\alpha^2 - 12\alpha + 2}$$

Ex 4) Multiply the following polynomials:

$$(5x^3 + 2x^2 + 5x + 2)$$

$$\cdot\ (2x^4 + 5x^3 + 10x^2 + 7)$$

Solution:

$(5x^3 + 2x^2 + 5x + 2) \cdot (2x^4 + 5x^3 + 10x^2 + 7)$

$= 5x^3(2x^4) + 5x^3(5x^3) + 5x^3(10x^2) + 5x^3(7) + 2x^2(2x^4) + 2x^2(5x^3) + 2x^2(10x^2) + 2x^2(7) + 5x(2x^4) + 5x(5x^3) + 5x(10x^2) + 5x(7) + 2(2x^4) + 2(5x^3) + 2(10x^2) + 2(7) =$

Polynomials (Part 1)

M. Kaabar

$= \boxed{10x^7} + 25x^6 + 50x^5 + 35x^3 + 4x^6 + 10x^5 + 20x^4 + 14x^2 + 10x^5 + 25x^4 + 50x^3 + 35x + 4x^4 + 10x^3 + 20x^2 + 14$

$= 10x^7 + 29x^6 + 70x^5 + 90x^3 + 49x^4 + 34x^2 + 35x + 14$

$= \boxed{10x^7 + 29x^6 + 70x^5 + 49x^4 + 90x^3 + 34x^2 + 35x + 14}$

Ex5) Divide the following polynomials using long division and synthetic division: (x^3-1) by $(x-1)$

To divide two polynomials say, A and B, such that $\frac{A(x)}{B(x)}$ where the degree of A is greater than or equal to the degree of B, we do the following:

1- Long Division.

2- Synthetic Division: It's only good if the B degree is 1 such as $B(x) = x -$ constant $\Rightarrow$

Solution

Long Division:

$$
\begin{array}{r|l}
 & x^2 + x + 1 \\
\hline
x - 1 & x^3 + 0x^2 + 0x - 1 \\
 & \ominus\, \cancel{+}x^3 \oplus \cancel{-}x^2 \\
\hline
 & x^2 + 0x - 1 \\
 & \ominus\, \cancel{+}x^2 \oplus \cancel{-}x \\
\hline
 & x - 1 \\
 & \ominus\, \cancel{+}x \oplus \cancel{-}1 \\
\hline
 & 0
\end{array}
$$

zero reminder

So,

$$\frac{x^3 - 1}{x - 1} = \boxed{x^2 + x + 1}$$

*let's solve it using synthetic division:

$$
\begin{array}{r|rrrr}
+1 & 1 & 0 & 0 & -1 \\
 & \downarrow & 1 & 1 & 1 \\
\hline
 & 1 & 1 & 1 & \boxed{0}
\end{array}
$$

$\rightarrow$ zero reminder

$\boxed{x^2 + x + 1}$ $\leftarrow$ result

which is the same one that we got above: