Study Guide 1
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MATH 140 Lab: Section 1
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Student's Name:------------------------------------------------------------
Student's ID: $\qquad$
Note: This study guide contains my practice questions that I think will be useful for preparing you for the first exam in Calculus for Life Scientists.

Question 1: Find the following limits. Show your work.
a. $\lim _{x \rightarrow 2}\left(\frac{x^{2}-5 x+6}{x^{2}-2 x}\right)$
b. $\lim _{x \rightarrow 1}\left(\frac{\sqrt{x}-x}{1-\sqrt{x}}\right)$
c. $\lim _{x \rightarrow 1} \frac{x-1}{\left|x-x^{2}\right|}$
d. $\lim _{x \rightarrow 0} \frac{3-3 \cos ^{2} x}{4 x^{2}}$
e. $\lim _{x \rightarrow-1^{+}} \frac{1-x}{(x+1)^{2}}$
f. $\lim _{x \rightarrow 2^{-}} \frac{x}{x-2}$
g. $\lim _{x \rightarrow \infty} \frac{2 x^{2}-1}{4 x^{3}-5 x-1}$
h. $\lim _{x \rightarrow 0^{+}} e^{\left(-\frac{1}{x}\right)}$
i. $\quad \lim _{x \rightarrow 0} \frac{\sin (x)+3 x}{2 x}$
j. $\lim _{x \rightarrow-\infty} \frac{2 x}{\sqrt{x^{2}+4}}$

Question 2: Find $f^{\prime}(1)$ and $h^{\prime}(2)$ using the definition of derivative where:

$$
f(x)=\frac{x}{x+1} \text { and } h(x)=\sqrt{x-1}
$$

Question 3: Discuss the continuity at $x=2$ for the following function:

$$
f(x)= \begin{cases}3, & x=2 \\ 3 x-2, & x>2 \\ x^{2}, & x<2\end{cases}
$$

Question 4: Find the equation of the tangent line to the curve: $y=4 \sqrt{x}-2 x$ at $x=4$.

Question 5: Find $y^{\prime}$ without simplifying your answer for the following:
a. $y=12 x-x^{2}-\frac{3}{\sqrt{x}}$
b. $y=x\left(3 x^{2}-\sqrt{x}\right)$
c. $y=\frac{2}{x^{4}}-x^{3}+2$
d. $y=\frac{x^{3}}{\left(x^{2}+4\right)^{2}}$
e. $y=e^{\sin (2 x)}$
f. $y=\ln \left(\sin \left(x^{2}\right)\right)$
g. $y=\left(x^{2}\right)^{x}$

Question 6: Find the equation of the tangent line to the curve: $y=\sin (4 x)$ at $x=\frac{\pi}{8}$.

Question 7: Consider the function: $g(x)= \begin{cases}1+m x^{2}, & x<1 \\ x^{2}+m x, & x \geq 1\end{cases}$
a. Show that $g(x)$ is continuous at $x=1$.
b. Find the value of $m$ for which $f(x)$ is differentiable at $x=1$.

Question 8: Find all points of discontinuity for the following functions
a. $h(x)= \begin{cases}\frac{x^{2}-3 x}{x-3}, & x \geq 3 \\ x+1, & x<3\end{cases}$
b. $\quad m(x)=\frac{3}{|2 x|+4}$

Question 9: Consider the function: $f(x)=\left\{\begin{array}{lr}\frac{1}{x^{3}+1}, & x<-1 \\ 2 x+1, & -1 \leq x<1 \\ 3 x^{2}, & 1<x \leq 2 \\ x^{3}, & x>2\end{array}\right.$
Find the following limits or state that the limit does exist. Explain why.
a. $\lim _{x \rightarrow-\infty} f(x)$
b. $\lim _{x \rightarrow 1} f(x)$
c. $\lim _{x \rightarrow-1} f(x)$
d. $\lim _{x \rightarrow 2} f(x)$

Question 10: Determine whether the following statements are true or false:
a. Assume that $f(x)=x^{2}-2 \sqrt{x}-1$ is defined on [1,2]. Then, there exists a number $c$ between 1 and 2 such that $f(c)=0$.
[...........................]
b. $\frac{d^{2} y}{d x^{2}}=\left(\frac{d y}{d x}\right)^{2}$
$[\ldots \ldots \ldots \ldots \ldots \ldots \ldots . . . . . . . . . . . . .$.
c. $y=1$ is a horizontal asymptote of $y=\frac{1}{x-1}$. $\quad[\ldots \ldots \ldots \ldots \ldots \ldots \ldots$.
d. If $y=x^{x}, y^{\prime}=x^{x}(\ln x+1)$.
[..........................]
e. $x=-1$ is a vertical asymptote of $y=\frac{x-2}{x^{2}-x-2}$.
[..........................]

Question 11: If $f(1)=3, f^{\prime}(1)=2, g(1)=10, g^{\prime}(1)=4, g^{\prime}(3)=-2$, then find the value of $w^{\prime}(1)$ where:
a. $\quad w(x)=g(f(x))$
b. $w(x)=x^{3} f(x)$
c. $w(x)=\ln \left(g(x)^{2}+1\right)$

Question 12: Find the equilibrium at $c_{1}=1$ for the following difference equation:

$$
c_{n+1}=\sqrt{c_{n}+2}
$$

Question 13*: Assume that the height of a falling object $t$ seconds after being dropped from a height of 64 feet can be written as follows:

$$
h(t)=64-16 t^{2} \text { feet }
$$

a. Find the average velocity between times $t=1$ and $t=2$.
b. Find the instantaneous velocity at times $t=2$.
*Reference: Calculus: Early Transcendental Functions by Smith Minton $3{ }^{\text {rd }}$ Edition Good Luck in Exam 1
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