Department of Mathematics
Moreno Valley College

Mathematics 52

Course ID: (27488)

Second Take-Home Midterm

Fall 2016

Dates: November 15th, 2016 and November 16th, 2016
Times: 8:00 AM – 10:05 AM and 2:00 PM – 4:05 PM

Professor: Mohammed Kaabar

P1	P2	P3	P4	P5	P6	P7	P8	P9	EC	Total
20	10	10	10	10	10	10	10	10	5	100

Student Name: Mohammed Kaabar

Student ID: – Solution –

Exam Instructions:

1- This exam has 8 questions and two extra credit questions.

2- Make sure you answer all questions.

3- Cheating = "F"

4- Make sure to include this page in your submission materials.

Student Signature:..................................

Problem 1 (20 points): Determine whether the following is **TRUE** or **FALSE** and if it is false **EXPLAIN** why:

False a. Linear inequality is a mathematical statement that has a mathematical expression that is greater than only. False / It has greater than or equal or smaller than or equal.

True b. The solution for $-5 + 7x < 3x + 7$ is $3 > x$.

False c. The solution for $\left(\frac{4z+5}{2} - \frac{1}{3}\right) \geq \left(-\frac{7}{2} + z\right)$ is $z \leq -\frac{34}{6}$.

solution is $z \geq \frac{-34}{6}$

False d. The general form of the interval notation can be written as $\{variable|solution\}$.

This is the set-builder notation

False e. (0,2) is located on the first quadrant only.

(0,2) is located between 1^{st} quadrant and 2^{nd} quadrant.

True f. $(-1,2)$ is located on the second quadrant.

False g. Given that l_1 and l_2 are non-vertical lines. If $l_1 \parallel l_2$, then $m_1 + m_2 = -1$.

If $l_1 // l_2$, then $m_1 = m_2$

True h. Given that l_1 and l_2 are non-vertical lines. If l_1 and l_2 make an angle of 90°, then $m_1 \cdot m_2 = -1$.

False i. It is impossible to derive the slope-point form of equation of line using the slope formula by considering the slope passes through $\overset{x_1\ y_1}{(x_1, y_1)}$ and $\overset{x_2\ y_2}{(x, y)}$.

It's possible $m = \frac{y - y_1}{x - x_1} \Rightarrow m(x - x_1) = y - y_1 \Rightarrow \boxed{y - y_1 = m(x - x_1)}$

False j. $y - intercept$ is defined as a point on the $y - axis$ that is considered the passing point for the graph of equation: $y = mx + b$ so the $y - interecept$ is $(b, 0)$.

y-intercept is (0, b)

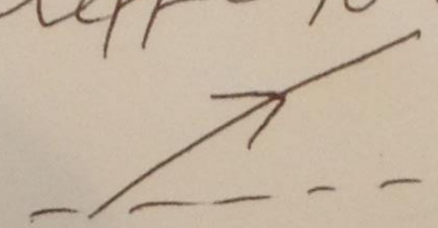
Problem 2 (10 points): Answer each of the following:

a. What is the name of zero slope? Horizontal slope

b. What is the name of undefined slope? Vertical slope

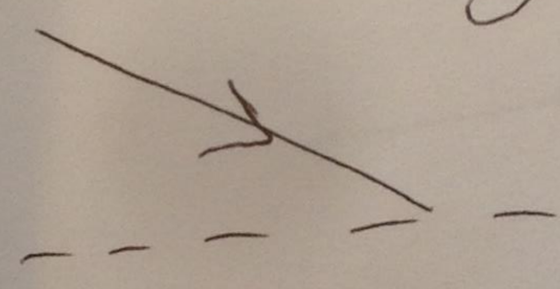
c. What is the positive slope? Rising as x moves from left to right.

d. Draw the positive slope:



e. What is the negative slope? Falling as x moves from left to right.

f. Draw the negative slope:



g. Derive the point-slope form of the equation of line:

Hint: Use (x_1, y_1) and (x, y) as two given points and write the slope formula $m = \frac{(y_2-y_1)}{(x_2-x_1)}$

let $x_1 = x_1$ $y_1 = y_1$

$x_2 = x$ $y_2 = y$

then, we find the slope of a line formula as follows:-

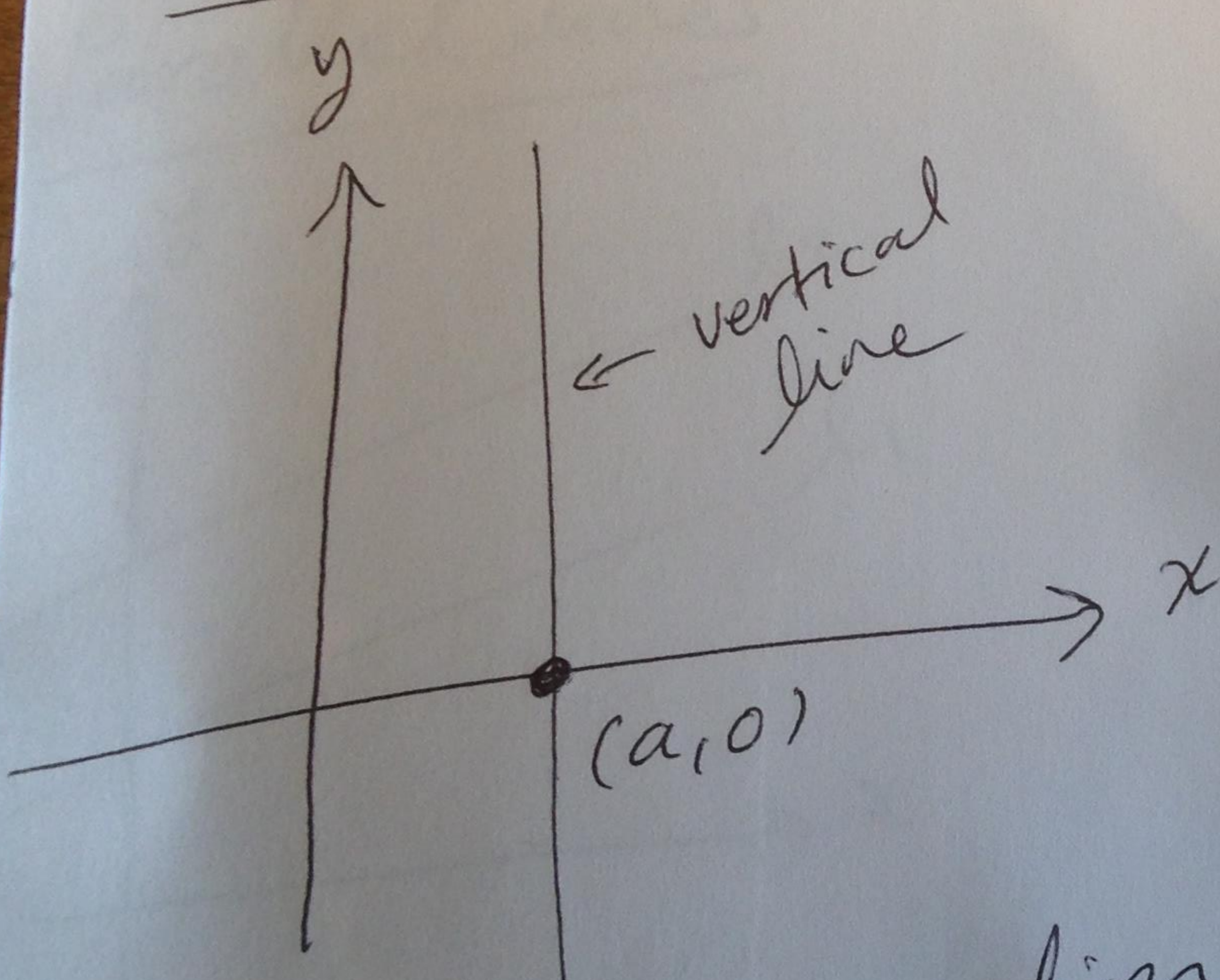
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$, now we do a cross multiplication, we obtain: $\boxed{y - y_1 = m(x - x_1)}$ point-slope form of equation of li

Problem 3 (10 points): In our class, we talked about two theorems of lines: vertical line and horizontal line. Discuss those two theorems and make sure to include examples and graphs for both lines.

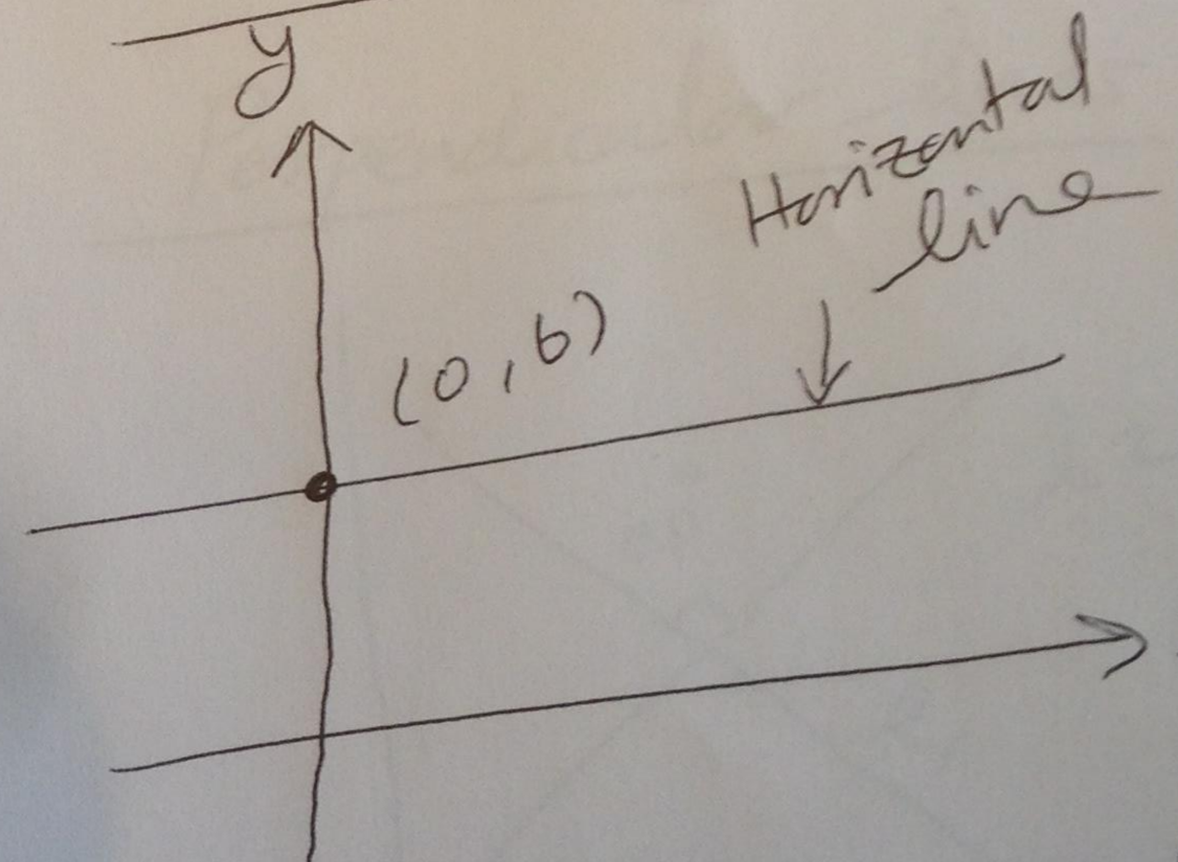
Hint: Use "Slope of a Line" lecture notes.

lines

Vertical line



Horizontal line



Theorem: the equation of vertical line passing through the point (a,0) is $\boxed{x = a}$.

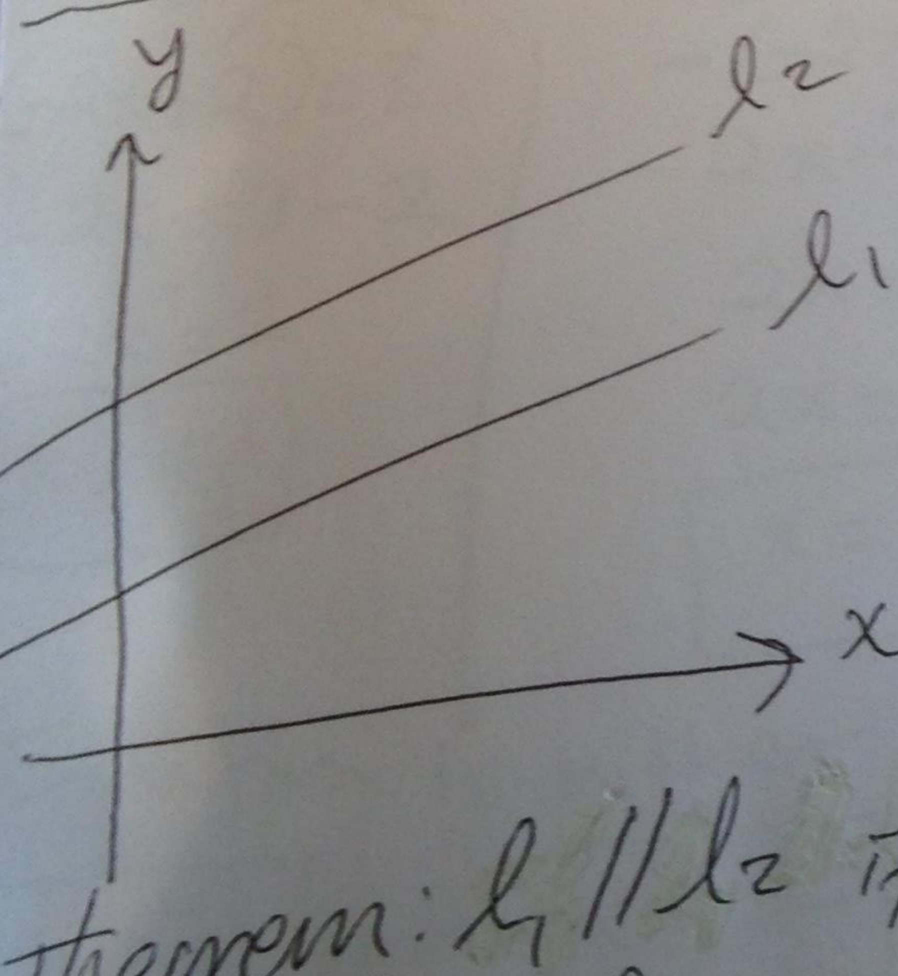
Theorem: the equation of horizontal line passing through the point (0,b) is $\boxed{y = b}$.

Problem 4 (10 points): In our class, we talked about two theorems of non-vertical lines: Discuss those two theorems and make sure to include examples and graphs for both non-vertical lines.

Hint: Use "Slope of a Line" lecture notes.

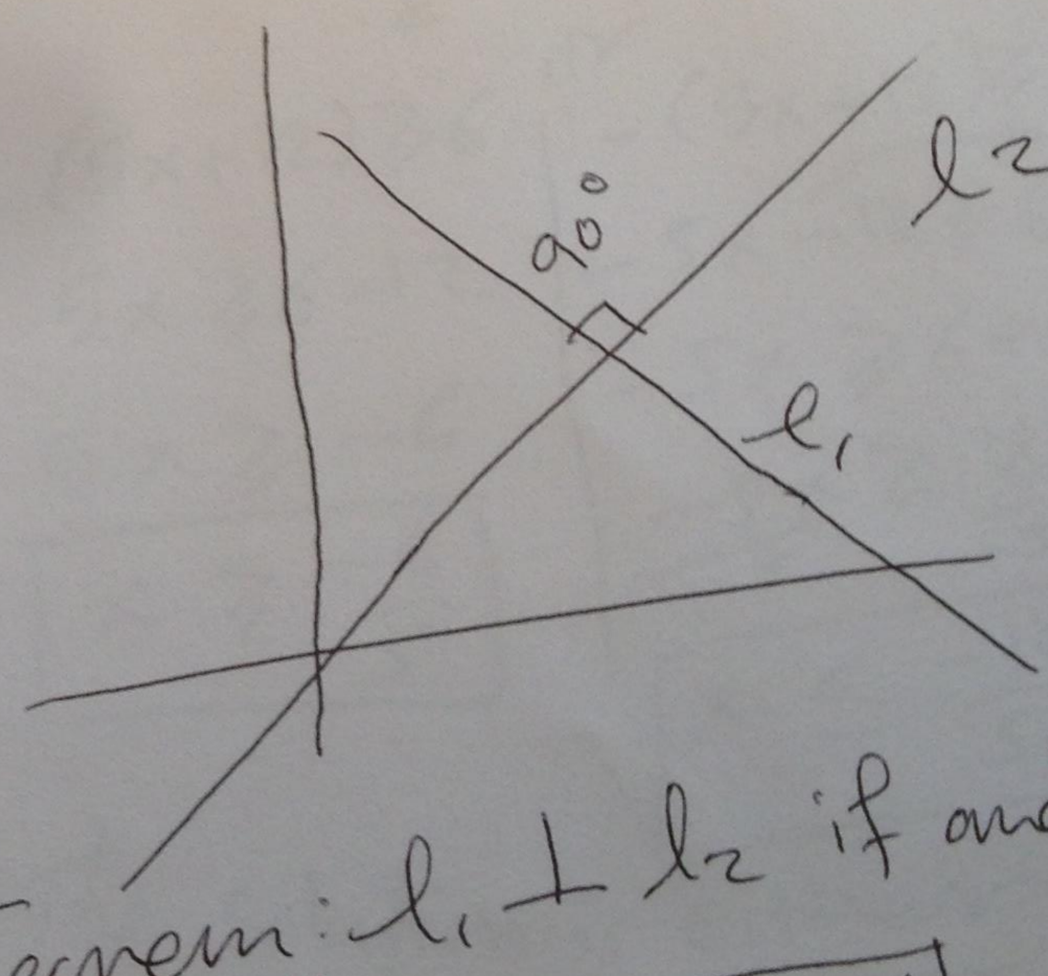
Non-Vertical lines

Parallel lines



Theorem: $l_1 \parallel l_2$ if and only if $\boxed{m_1 = m_2}$

Perpendicular lines



Theorem: $l_1 \perp l_2$ if and only if $\boxed{m_1 \cdot m_2 = -1}$

Problem 5 (10 points): Solve **TWO** of the following **FIVE** problems:

1- Solve for x given that $|-2x+2| = 3$.

2- Solve for x given that $|5x+12| \geq 6$.

3- A line passes through $(2,-1)$ and it is perpendicular to another line: $2y + 3 - 5y = -2x + 5x$. Write the equation for this line.

4- Solve the following linear inequality:

$$15\beta + \sqrt[3]{8} < (-6766776.766)^0 + 2\beta$$

5- Solve the following linear inequality:

$$-2\beta + 1^{\sqrt[3]{8}} < \left(-\frac{-23433.63}{-343544.12}\right)^0 + 12\beta$$

1 $|-2x+2| = 3$

$(-2x+2) = 3$

$-2x = 3-2$

$-2x = 1$

$\boxed{x = \frac{-1}{2}}$

or

$-(-2x+2) = 3$

$2x - 2 = 3$

$2x = 3+2$

$2x = 5$

$\boxed{x = \frac{5}{2}}$

Solution is either $\boxed{x = \frac{-1}{2}}$ or $\boxed{x = \frac{5}{2}}$

2 $|5x+12| \geq 6$

$(5x+12) \geq 6$

$5x \geq 6-12$

$5x \geq -6$

$\boxed{x \geq \frac{-6}{5}}$

or

$-(5x+12) \geq 6$

$-5x - 12 \geq 6$

$-5x \geq 6+12$

$\frac{-5x}{-5} \geq \frac{18}{-5}$

$\boxed{x \leq \frac{-18}{5}}$

The solution is either $\boxed{x \geq \frac{-6}{5}}$ or $\boxed{x \leq \frac{-18}{5}}$

6-a

Continue problem 5 solution :-

3 $\quad \overset{l_1}{} (\overset{x_1}{2}, \overset{y_1}{-1})$

$l_2: \; 2y + 3 - 5y = -2x + 5x$

$\Rightarrow -3y + 3 = +3x$

$\Rightarrow -3y = 3x - 3$

$\Rightarrow \dfrac{-\cancel{3}y}{-\cancel{3}} = \dfrac{3x-3}{-3} \Rightarrow \boxed{y = -x + 1}$

So, $\boxed{m_2 = -1}$

Since $l_1 \perp l_2$, then $m_1 \cdot m_2 = -1$

$\boxed{?} \cdot -1 = -1$

So, $\boxed{m_1 = 1}$

Hence, $y - y_1 = m(x - x_1)$

$y + 1 = 1(x - 2)$

$y + 1 = x - 2$

$y = x - 2 - 1$

$\boxed{y = x - 3} \leftarrow$ the equation of line

Continue Problem ⑤ Solution:-

④ $15\beta + \sqrt[3]{8} < (-67667 76.766)^0 + 2\beta$

$\Rightarrow 15\beta + 2 < 1 + 2\beta$

$\Rightarrow 15\beta - 2\beta < 1 - 2$

$\Rightarrow \frac{\cancel{13}\beta}{\cancel{13}} < \frac{-1}{13} \Rightarrow \boxed{\beta < \frac{-1}{13}}$

This is the solution

⑤ $-2\beta + 1^{\sqrt[3]{8}} < \left(-\frac{-23433.63}{-343544.12}\right)^0 + 12\beta$

$\Rightarrow -2\beta + 1^2 < 1 + 12\beta$

$\Rightarrow -2\beta + 1 < 1 + 12\beta$

$\Rightarrow 1 - 1 < 12\beta + 2\beta$

$\Rightarrow \frac{\overset{0}{\cancel{0}}}{\cancel{14}} < \frac{\cancel{14}\beta}{\cancel{14}} \Rightarrow \boxed{0 < \beta}$

This is the solution

Problem 6 (10 points): Discussion Problems:

a. When we talked about dividing the polynomials, we mentioned that there are two methods of division: long and synthetic division. In addition, we talked about a common property for both of them and a limited property for synthetic division only. Discuss that in more details.

*Common Property: The degree of polynomial is the numerator is greater than or equal to the degree of polynomial in the denominator

Limited Property: The denominator has to be degree 1, and it's written as

$\boxed{\text{variable} \pm \text{constant}}$.

b. We talked about the properties of factoring, and I asked a question: Given that a and b are real numbers, then Is $(a-b)^2 = a^2 - b^2$???!!! Discuss that in more details.

No, to prove that, find $(a-b)^2$ and a^2-b^2 individually as follows:

$(a-b)^2 = a^2 - 2ab + b^2$

$(a^2-b^2) = (a-b)(a+b)$

$a^2 - 2ab + b^2 \neq (a-b)(a+b)$

or

Hence, $(a-b)^2 \neq (a^2-b^2)$

Problem 7 (10 points): Use either long division or synthetic division to do the following:

$$\frac{x^3 + x^2 - x - 1}{x - 3}$$

$x^3 + x^2 - x - 1 \rightarrow$ call it $A(x)$

$x - 3 \rightarrow$ call it $B(x)$

Check:- $\deg(A(x)) \geq \deg(B(x))$

$3 \geq 1$ So, we can use long division.

$$\boxed{x^2 + 4x + 11}$$

$$x - 3 \overline{\smash{)}\, x^3 + x^2 - x - 1}$$

$\ominus\ \cancel{x^3} \oplus 3x^2$

$4x^2 - x - 1$

$\ominus\ \cancel{4x^2} \oplus 12x$

$11x - 1$

$\ominus\ \cancel{11x} \oplus 33$

$\boxed{32} \leftarrow$ remainder

Synthetic Division

$$\begin{array}{r|rrrr} 3 & 1 & 1 & -1 & -1 \\ & \downarrow & 3 & 12 & 33 \\ \hline & 1 & 4 & 11 & \boxed{32} \end{array}$$

Solution: $(x^2 + 4x + 11) + \frac{32}{(x-3)}$ remainder

same

the solution is: $\left(\boxed{(x^2 + 4x + 11)} + \frac{\boxed{32}}{\boxed{(x-3)}} \right)$

Problem 8 (10 points): Factor each of the following:

a. $(x^2 - 12)$

$(x^2 - 12) = (x - \sqrt{12})(x + \sqrt{12})$

b. $(x - 25)$

$(x - 25) = (\sqrt{x} - 5)(\sqrt{x} + 5)$

c. $(16a^2 - 48ac + 36c^2 - 100) = 4(4a^2 - 12ac + 9c^2 - 25)$

$= 4((2a - 3c)^2 - 25)$

$= 4(((2a-3c) - 5)((2a-3c)+5))$

d. $(25x^2 - 16)$

$= (5x - 4)(5x + 4)$

e. $(24z^2 - 12)$

$= 4(6z^2 - 3)$

$= 4(\sqrt{6}z - \sqrt{3})(\sqrt{6}z + \sqrt{3})$

or $= (\sqrt{24}z - \sqrt{12})(\sqrt{24}z + \sqrt{12})$

or $= 2(12z^2 - 6)$

$= 2(\sqrt{12}z - \sqrt{6})(\sqrt{12}z + \sqrt{6})$

Problem 9 (10 points): Simplify each of the following:

a. $(2x-1)^2 = \boxed{4x^2 - 4x + 1}$

b. $x^3y^{-1}z^2m^2ym^{-2}x^{-2} = \dfrac{x^{\cancel{3}\,1} \cancel{y} z^2 \cancel{m^2}}{\cancel{x^2}\,\cancel{y}\,\cancel{m^2}} = \boxed{xz^2}$

c. $\left(-\frac{x^3}{3y^2x^7}\right)^3 = \left(-\dfrac{x^{-4}}{3y^2}\right)^3 = \left(-\dfrac{1}{3x^4y^2}\right)^3 = \boxed{\dfrac{-1}{27x^{12}y^6}}$

d. $(x^2+1)^2 = \boxed{x^4 + 2x^2 + 1}$

e. $7y^2x^3(-3x^{-3}y^{-5}) = -21\left(\dfrac{\cancel{y^2}\,\cancel{x^3}}{y^{\cancel{5}\,3}\,\cancel{x^3}}\right) = \dfrac{-21}{y^3} = \boxed{-21y^{-3}}$

<u>Extra Credit Problem (5 points):</u> Use **<u>only synthetic division</u>** to do the following:

$$\frac{x^3 + x^2 - x - 1}{2x^2 - x + 2}$$

No solution because we cannot use synthetic division here since the denominator is not written on the form: $\boxed{\text{variable} \pm \text{constant}}$

I wish you best of luck in Exam 2

Best Regards

Professor: Mohammed Kaabar