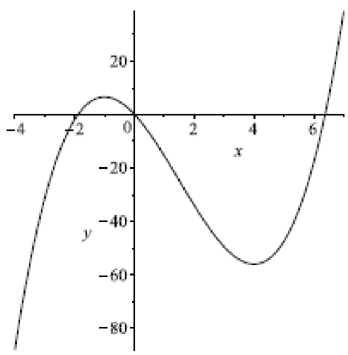
Assignment 8 (SOLUTION from Textbook Manual Solution)

Text: *Calculus for the Life Sciences, S. Schreiber, K. Smith and W. Getz, Wiley, 2014*

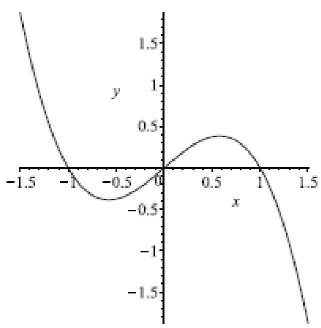
Section 4.1

2. The function is defined and continuous everywhere, thus there are no vertical asymptotes. $\lim_{x\to\pm\infty}(x^2+5x-3)=\lim_{x\to\pm\infty}x(x+5)-$

7. The function is defined and continuous everywhere, thus there are no vertical asymptotes. $\lim_{x\to\pm\infty}(-12x-9x^2/2+x^3)=\pm\infty$, so there are no horizontal asymptotes. $\lim_{x\to\pm\infty}y/x=\lim_{x\to\pm\infty}(x^2-9x/2-12)=\infty$, so there are no linear asymptotes. $y'=-12-9x+3x^2=3(x-4)(x+1)$, $y''=-9+6x$. The function is decreasing when $y'<0$, i.e. when $-1<x<4$ and increasing when $y'>0$, i.e. when $x<-1$ and $x>4$; it is concave up when $y''>0$, i.e. when $x>3/2$ and concave down when $y''<0$, i.e. when $x<3/2$; there is an inflection point at $x=3/2$.



11. The function is defined and continuous everywhere, thus there are no vertical asymptotes. $\lim_{x\to\pm\infty}(x - x^3) = \mp\infty$, so there are no horizontal asymptotes. $\lim_{x\to\pm\infty} y/x = \lim_{x\to\pm\infty}(1 - x^2) = -\infty$, so there are no linear asymptotes. $y' = 1 - 3x^2$, $y'' = -6x$. The function is decreasing when $y' < 0$, i.e. when $x < -1/\sqrt{3}$ and $x > 1/\sqrt{3}$ and increasing when $y' > 0$, i.e. when $-1/\sqrt{3} < x < 1/\sqrt{3}$; it is concave up when $y'' > 0$, i.e. when $x < 0$ and concave down when $y'' < 0$, i.e. when $x > 0$; there is an inflection point at $x = 0$.



Section 4.2

17. $f'(x) = 2x - 4$; the only critical point is $x = 2$, which is inside the given interval. $f(0) = 2$, $f(3) = -1$ and $f(2) = -2$. Thus on $[0, 3]$, the global minimum is $f = -2$ at $x = 2$ and the global maximum is $f = 2$ at $x = 0$.

18. $f'(x) = 3x^2 - 12$; the critical points are $x = -2$ and $x = 2$, both inside the given interval. $f(-3) = 11$, $f(3) = -7$, $f(2) = -14$

and $f(-2) = 18$. Thus on $[-3, 3]$, the global minimum is $f = -14$ at $x = 2$ and the global maximum is $f = 18$ at $x = -2$.

20. $f'(x) = e^{-x} - xe^{-x} = (1 - x)e^{-x}$; the only critical point is $x = 1$ which is inside the given interval. $f(0) = 0$, $f(100) = 100/e^{100}$, and $f(1) = 1/e$. Thus on $[0, 100]$, the global minimum is $f = 0$ at $x = 0$ and the global maximum is $f = 1/e$ at $x = 1$.

22. $f'(x) = 3x^2 - 12$; critical point on $(0, \infty)$ is $x = 2$. $\lim_{x\to\infty}(x^3 - 12x + 2) = \infty$, $\lim_{x\to 0^+}(x^3 - 12x + 2) = 2$. This implies that the global minimum is $f = -14$ at $x = 2$ and there is no global maximum.

24. $f'(x) = e^{-x} - xe^{-x} = (1 - x)e^{-x}$; the only critical point is $x = 1$. $\lim_{x\to\infty}(xe^{-x}) = 0$ and $\lim_{x\to\infty}(xe^{-x}) = -\infty$, thus the global maximum is $f = 1/e$ at $x = 1$ and there is no global minimum.

Section 4.3

25. a. Using the notations and results of Example 4, if the doubling time is T, then $a = \ln 2/T$, and if the half life is S, then $b = \ln(1/2)/S$. Thus $V(t) = 0.44(0.9973e^{\ln(1/2)/6.24t} + 0.0027e^{\ln 2/2.9t}) = 0.4388e^{-0.111t} + 0.0012e^{0.239t}$.

b. $V' = -0.0487e^{-0.111t} + 0.00029e^{0.239t}$, thus $V' = 0$ at $t \approx 14.697$ days.

c. The model overestimates the time by about a day.

Section 5.1

3. We obtain $F(x) = x^2 + 3x + C$, because $F'(x) = 2x + 3$ by the power rule.